

## Exercise 18

Find the limit.

$$\lim_{x \rightarrow \infty} e^{x-x^2}$$

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### Solution

Evaluate this limit by writing the difference as a quotient. Note that since the exponential function is continuous, the limit can be brought inside.

$$\begin{aligned}\lim_{x \rightarrow \infty} e^{x-x^2} &= \exp \left[ \lim_{x \rightarrow \infty} (x - x^2) \right] \\ &= \exp \left[ \lim_{x \rightarrow \infty} (x - x^2) \times \frac{x + x^2}{x + x^2} \right] \\ &= \exp \left[ \lim_{x \rightarrow \infty} \frac{(x - x^2)(x + x^2)}{x + x^2} \right] \\ &= \exp \left( \lim_{x \rightarrow \infty} \frac{x^2 - x^4}{x + x^2} \right) \\ &= \exp \left[ \lim_{x \rightarrow \infty} \frac{x^2(1 - x^2)}{x^2 \left( \frac{1}{x} + 1 \right)} \right] \\ &= \exp \left( \lim_{x \rightarrow \infty} \frac{1 - x^2}{\frac{1}{x} + 1} \right) \\ &= \exp \left( \frac{1 - \infty}{0 + 1} \right) \\ &= e^{-\infty} \\ &= 0\end{aligned}$$